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Quantum cosmology, inflationary brane-world creation and dS/CFT correspondence

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Abstract: The creation of 4d de Sitter (inflationary) boundary gluing two d5 de Sitter bulks on the classical as well as on quantum level (with account of brane QFT via corresponding trace anomaly induced effective action) is discussed. Quantum effects decrease the classical de Sitter brane radius or create new de Sitter brane with even smaller radius. It is important that brane CFT may be chosen to be dual to one of 5d de Sitter bulks, making the explicit relation of de Sitter brane-world with dS/CFT correspondence. In this way, the localization of gravity on the brane is shown. Moving (time-dependent) de Sitter brane in d5 SdS BH is considered. In the special coordinate system where brane equations look like quantum-corrected FRW equations the comparison with similar brane equations in SAdS BH bulk is done.

Keywords: D-branes, AdS-CFT Correspondence, Black Holes.

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Contents

1. Introduction

There are various ways to realize the Randall-Sundrum brane-world Universe [1]. In particulary, having in mind the relation with AdS/CFT correspondence [2] in refs. [3, 4] (for related discussion, see [5]), the quantum creation of the brane-world thanks to conformal anomaly of four-dimensional fields has been discussed. The mechanism of refs. [3, 4] has been applied to construct the (inflationary) Brane New World.

Recently much attention has been paid to dS/CFT correspondence [6, 9] which is similar in spirit to AdS/CFT correspondence. (For earlier proposals on dS/CFT duality, see [7] and for recent related discussion of thermodynamics in de Sitter space, see [8]). The reason why AdS/CFT can be expected is the isometry of $d+1$-dimensional anti-de Sitter space, which is $\mathrm{SO}(d,2)$ symmetry. It is identical with the conformal symmetry of d-dimensional Minkowski space. We should note, however, $d+1$-dimensional de Sitter space has the isometry of $\mathrm{SO}(d+1,1)$ symmetry, which can be a conformal symmetry of d-dimensional euclidean space. Then it might be natural to expect the correspondence between $d+1$-dimensional de Sitter space and d-dimensional euclidean conformal symmetry (dS/CFT correspondence [6, 9]). In fact, the metric of $D=d+1$-dimensional anti de Sitter space (AdS) is given by

$$ds^2_{\mathrm{AdS}} = dr^2 + \mathrm{e}^{2r}\left(-dt^2 + \sum_{i=1}^{d-1}\left(dx^i\right)^2\right). \tag{1.1}$$

In the above expression, the boundary of AdS lies at $r=\infty$. If one exchanges the radial coordinate r and the time coordinate t, we obtain the metric of the de Sitter space (dS):

$$ds^2_{\mathrm{dS}} = -dt^2 + \mathrm{e}^{2t}\sum_{i=1}^{d}\left(dx^i\right)^2 . \tag{1.2}$$

Here $x^d = r$. Then there is a boundary at $t=\infty$, where the euclidean conformal field theory (CFT) can live and one expects dS/CFT correspondence as one more manifestation of holographic principle. This may be very important as there are indications that our Universe has de Sitter phase in the past and in the future. Then, there appears very nice way to formulate some de Sitter gravitational physics in terms of the boundary QFT physics and vice-versa.

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The purpose of the present paper is to consider the possibility of quantum creation of the inflationary brane in de Sitter bulk space in frames of mechanism of refs. [3, 4]. Note that such approach represents the generalization of so-called anomaly-driven inflation [15]. Moreover, the content of quantum fields on the brane may be chosen in such a way, that it corresponds to euclidean CFT dual to 5d dS bulk space. In this sense, one can understand that quantum creation of dS brane-world occurs in frames of dS/CFT correspondence. In [4] several cases corresponding to flat, sphere or hyperboloid (brane) embedded in 5d AdS space have been considered. If we Wick-rotate 5-dimensional de Sitter space into the euclidean signature, we obtain 5d sphere. Then one cannot embedd the 4d flat or hyperbolic brane in the bulk 5d space. That is why we consider only the case that brane is 4d sphere.

We will show that gravity on such de Sitter brane (despite the fact that bulk represents not AdS but dS space) may be localized using proposed dS/CFT correspondence. Then, our model may be understood as four-dimensional gravity coupled to some gauge theory. As a result the model turns out to some kind of trace-anomaly driven inflation, which is known [18] may become instable. As a result there is natural solution to end the inflationary phase. The equations of moving brane in SdS background are also considered and presented as FRW equations with quantum corrections.

2. de Sitter brane-worlds

The metric of 5 dimensional euclidean de Sitter space that is 5d sphere is given by

$$ds_{\mathrm{S}_5}^2 = dy^2 + l^2 \sin^2 \frac{y}{l} d\Omega_4^2 \,. \tag{2.1}$$

Here $d\Omega_4^2$ describes the metric of S_4 with unit radius. The coordinate y is defined in $0 \leq y \leq l\pi$. One also assumes the brane lies at $y = y_0$ and the bulk space is given by gluing two regions given by $0 \leq y < y_0$.

We start with the action S which is the sum of the Einstein-Hilbert action S_{EH} with positive cosmological constant, the Gibbons-Hawking surface term S_{GH}, the surface counter term S_1[1] and the trace anomaly induced action $\mathcal{W}$:

$$\begin{aligned}
S &= S_{\mathrm{EH}} + S_{\mathrm{GH}} + 2S_1 + \mathcal{W} \,, \qquad S_{\mathrm{EH}} = \frac{1}{16\pi G}\int d^5x\sqrt{g_{(5)}}\left(R_{(5)} - \frac{12}{l^2}\right) \,, \\
S_{\mathrm{GH}} &= \frac{1}{8\pi G}\int d^4x\sqrt{g_{(4)}}\,\nabla_\mu n^\mu \,, \qquad S_1 = -\frac{3}{8\pi G l}\int d^4x\sqrt{g_{(4)}} \,, \\
\mathcal{W} &= b\int d^4x\sqrt{\widetilde{g}}\,\widetilde{F}A + b'\int d^4x\sqrt{\widetilde{g}}\left\{A\left[2\widetilde{\Box}^2 + \widetilde{R}_{\mu\nu}\widetilde{\nabla}_\mu\widetilde{\nabla}_\nu - \frac{4}{3}\widetilde{R}\,\widetilde{\Box}^2 + \frac{2}{3}(\widetilde{\nabla}^\mu\widetilde{R})\widetilde{\nabla}_\mu\right]A + \right. \\
&\qquad\qquad \left. + \left(\widetilde{G} - \frac{2}{3}\widetilde{\Box}\widetilde{R}\right)A\right\} - \\
&\qquad - \frac{1}{12}\left\{b'' + \frac{2}{3}(b + b')\right\}\int d^4x\sqrt{\widetilde{g}}\left[\widetilde{R} - 6\,\widetilde{\Box}A - 6(\widetilde{\nabla}_\mu A)(\widetilde{\nabla}^\mu A)\right]^2 \,.
\end{aligned} \tag{2.2}$$

[1] The coefficient of S_1 cannot be determined from the condition of finiteness of the action as in AdS/CFT. However, using the renormailzation group method as in [10] this coefficient can be determined uniquely, see also third paper in [9].

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Here the quantities in the 5 dimensional bulk spacetime are specified by the suffices $_{(5)}$ and those in the boundary 4 dimensional spacetime by $_{(4)}$. The factor 2 in front of S_1 in (2.2) is coming from that we have two bulk regions which are connected with each other by the brane. In (2.2), n^μ is the unit vector normal to the boundary. In (2.2), one chooses the 4 dimensional boundary metric as

$$g_{(4)\mu\nu} = \mathrm{e}^{2A}\tilde{g}_{\mu\nu} \tag{2.3}$$

and we specify the quantities with $\tilde{g}_{\mu\nu}$ by using $\tilde{}$. G ($\tilde{G}$) and F ($\tilde{F}$) are the Gauss-Bonnet invariant and the square of the Weyl tensor, which are given as[2]

$$G = R^2 - 4R_{ij}R^{ij} + R_{ijkl}R^{ijkl}\,, \qquad F = \frac{1}{3}R^2 - 2R_{ij}R^{ij} + R_{ijkl}R^{ijkl}\,, \tag{2.4}$$

In the effective action (2.9) induced by brane quantum matter, in general, with N real scalar, $N_{1/2}$ Dirac spinor, N_1 vector fields, N_2 ($= 0$ or 1) gravitons and $N_{\rm HD}$ higher derivative conformal scalars, b, b' and b'' are

$$\begin{aligned} b &= \frac{N + 6N_{1/2} + 12N_1 + 611N_2 - 8N_{\rm HD}}{120(4\pi)^2}\,, \\ b' &= -\frac{N + 11N_{1/2} + 62N_1 + 1411N_2 - 28N_{\rm HD}}{360(4\pi)^2}\,, \qquad b'' = 0\,. \end{aligned} \tag{2.5}$$

Usually, b'' may be changed by the finite renormalization of local counterterm in the gravitational effective action but as we will see later, the term proportional to $\left\{b'' + \frac{2}{3}(b + b')\right\}$ in (2.9), and therefore b'' does not contribute to the equations describing the nucleation of the brane. Nevetheless, this parameter plays an important role in tensor perturbations, what leads to decay of de Sitter space (end of inflation).

For typical examples motivated by AdS/CFT (and presumbly by dS/CFT because central charges are the same in AdS/CFT or dS/CFT) correspondence one has: (a) $\mathcal{N} = 4$ SU(N) SYM theory $b = -b' = \frac{N^2-1}{4(4\pi)^2}$, ($b$) $\mathcal{N} = 2$ Sp(N) theory $b = \frac{12N^2+18N-2}{24(4\pi)^2}$, $b' = -\frac{12N^2+12N-1}{24(4\pi)^2}$. Note that b' is negative in the above cases.

We should also note that $\mathcal{W}$ in (2.9) is defined up to conformally invariant functional, which cannot be determined from only the conformal anomaly. The conformally flat space is a pleasant exclusion where anomaly induced effective action is defined uniquely. However, one can argue that such conformally invariant functional gives next to leading contribution as mass parameter of regularization may be adjusted to be arbitrary small (or large).

The metric of S_4 with the unit radius is given by

$$d\Omega_4^2 = d\chi^2 + \sin^2\chi d\Omega_3^2\,. \tag{2.6}$$

[2]We use the following curvature conventions:

$$\begin{aligned} R &= g^{\mu\nu}R_{\mu\nu}\,, & R_{\mu\nu} &= R^\lambda{}_{\mu\lambda\nu} \\ R^\lambda{}_{\mu\rho\nu} &= -\Gamma^\lambda_{\mu\rho,\nu} + \Gamma^\lambda_{\mu\nu,\rho} - \Gamma^\eta_{\mu\rho}\Gamma^\lambda_{\nu\eta} + \Gamma^\eta_{\mu\nu}\Gamma^\lambda_{\rho\eta}\,, & \Gamma^\eta_{\mu\lambda} &= \frac{1}{2}g^{\eta\nu}\left(g_{\mu\nu,\lambda} + g_{\lambda\nu,\mu} - g_{\mu\lambda,\nu}\right). \end{aligned}$$

Here $d\Omega_3^2$ is described by the metric of 3 dimensional unit sphere. If one changes the coordinate χ to σ by $\sin\chi = \pm 1/\cosh\sigma$, one obtains[3]

$$d\Omega_4^2 = \frac{1}{\cosh^2\sigma}\left(d\sigma^2 + d\Omega_3^2\right) . \tag{2.7}$$

Then one assumes the metric of 5 dimensional space time as follows:

$$ds^2 = dy^2 + \mathrm{e}^{2A(y,\sigma)}\tilde g_{\mu\nu}dx^\mu dx^\nu , \qquad \tilde g_{\mu\nu}dx^\mu dx^\nu \equiv l^2\left(d\sigma^2 + d\Omega_3^2\right) \tag{2.8}$$

and one identifies A and $\tilde g$ in (2.8) with those in (2.3). Then $\tilde F = \tilde G = 0$, $\tilde R = 6/l^2$ etc. Due to the assumption (2.8), the actions in (2.2) have the following forms:

$$\begin{aligned}
S_{\rm EH} &= \frac{l^4 V_3}{16\pi G}\int dy d\sigma\Big\{\left(-8\partial_y^2 A - 20(\partial_y A)^2\right)\mathrm{e}^{4A} + \\
&\qquad\qquad + \left(-6\partial_\sigma^2 A - 6(\partial_\sigma A)^2 + 6\right)\mathrm{e}^{2A} - \frac{12}{l^2}\mathrm{e}^{4A}\Big\} \\
S_{\rm GH} &= \frac{l^4 V_3}{2\pi G}\int d\sigma \mathrm{e}^{4A}\partial_y A , \qquad S_1 = -\frac{3l^3 V_3}{8\pi G}\int d\sigma \mathrm{e}^{4A} \\
\mathcal{W} &= V_3\int d\sigma\left[b' A\left(2\partial_\sigma^4 A - 8\partial_\sigma^2 A\right) - 2(b+b')\left(1 - \partial_\sigma^2 A - (\partial_\sigma A)^2\right)^2\right] .
\end{aligned} \tag{2.9}$$

Here V_3 is the volume or area of the unit 3 sphere.

In the bulk, one obtains the following equation of motion from $S_{\rm EH}$ by the variation over A:

$$0 = \left(-24\partial_y^2 A - 48(\partial_y A)^2 - \frac{48}{l^2}\right)\mathrm{e}^{4A} + \frac{1}{l^2}\left(-12\partial_\sigma^2 A - 12(\partial_\sigma A)^2 + 12\right)\mathrm{e}^{2A} , \tag{2.10}$$

which corresponds to one of the Einstein equations. Then one finds solutions, A_S, which correspond to the metric dS_5 in (2.1) with (2.7).

$$A = A_S = \ln\sin\frac{y}{l} - \ln\cosh\sigma . \tag{2.11}$$

On the brane at the boundary, one gets the following equation:

$$\begin{aligned}
0 &= \frac{48 l^4}{16\pi G}\left(\partial_y A - \frac{1}{l}\right)\mathrm{e}^{4A} + b'\left(4\partial_\sigma^4 A - 16\partial_\sigma^2 A\right) - \\
&\quad - 4(b+b')\left(\partial_\sigma^4 A + 2\partial_\sigma^2 A - 6(\partial_\sigma A)^2\partial_\sigma^2 A\right) .
\end{aligned} \tag{2.12}$$

We should note that the contributions from $S_{\rm EH}$ and $S_{\rm GH}$ are twice from the naive values since we have two bulk regions which are connected with each other by the brane. Substituting the bulk solution $A = A_S$ in (2.11) into (2.12) and defining the radius R of the brane by $R \equiv l\sin\frac{y_0}{l}$, one obtains

$$0 = \frac{1}{\pi G}\left(\frac{1}{R}\sqrt{1 - \frac{R^2}{l^2}} - \frac{1}{l}\right)R^4 + 8b' . \tag{2.13}$$

[3]If we Wick-rotate the metric by $\sigma \to it$, we obtain the metric of de Sitter space:

$$d\Omega_4^2 \to ds_{\rm dS}^2 = \frac{1}{\cos^2 t}\left(-dt^2 + d\Omega_3^2\right) .$$

One sees that eq. (2.13) does not depend on b. First we should note $0 \leq R \leq l$ by definition. Even in the case that there is no quantum contribution from the matter on the brane, that is, $b' = 0$, eq. (2.13) has a solution:

$$R^2 = R_0^2 \equiv \frac{l^2}{2} \text{ or } \frac{y_0}{l} = \frac{\pi}{4}, \frac{3\pi}{4}. \tag{2.14}$$

In eq. (2.13), the first term $\frac{R^3}{\pi G}\sqrt{1 - R^2/l^2}$ corresponds to the gravity, which makes the radius R larger. On the other hand, the second term $-R^4/\pi G l$ corresponds to the tension, which makes R smaller. When $R < R_0$, gravity becomes larger than the tension and when $R > R_0$, vice versa. Then both of the solutions in (2.14) are stable. Although it is not clear from (2.13), $R = l$ ($y/l = \pi/2$) corresponds to the local maximum. Hence, the possibility of creation of inflationary brane in de Sitter bulk is possible already on classical level, even in situation when brane tension is fixed by holographic RG. That is qualitatively different from the case of AdS bulk where only quantum effects led to creation of inflationary brane [4, 3] (when brane tension was not free parameter).

Let us make several remarks about properties of dS brane-world. There is an excellent explanation [3] why gravity is trapped on the brane in the AdS spacetime. This uses $\mathrm{AdS}_5/\mathrm{CFT}_4$ correspondence and the surface counter terms. This can be generalized to the brane in dS spacetime by using proposed dS/CFT correspondence.

In [16] it has been shown that the bulk action diverges in de Sitter space when we substitute the classical solution, which is the fluctuation around the de Sitter space in (1.2). In other words, counterterms are necessary again. The divergence occurs since the volume of the space diverges when $t \to \infty$ (or $t \to -\infty$ after replacing t by $-t$ in another patch). Then we should put the counterterms on the space-like branes which lie at $t \to \pm\infty$. Therefore dS/CFT correspondence should be given by

$$\begin{aligned}
\mathrm{e}^{-W_{\mathrm{CFT}}} &= \int [dg][d\varphi]\mathrm{e}^{-S_{\mathrm{dS\,grav}}}, \quad S_{\mathrm{dS\,grav}} = S_{\mathrm{EH}} + S_{\mathrm{GH}} + S_1 + S_2 + \cdots, \\
S_{\mathrm{EH}} &= \frac{1}{16\pi G}\int d^5x\sqrt{-g_{(5)}}\left(R_{(5)} - \frac{12}{l^2} + \cdots\right), \\
S_{\mathrm{GH}} &= \frac{1}{8\pi G}\int_{M_4^+ + M_4^-} d^4x\sqrt{g_{(4)}}\,\nabla_\mu n^\mu, \\
S_1 &= \frac{3}{8\pi G l}\int_{M_4^+ + M_4^-} d^4x\sqrt{g_{(4)}}, \qquad S_2 = \frac{l}{32\pi G}\int_{M_4^+ + M_4^-} d^4x\sqrt{g_{(4)}}\left(R_{(4)} + \cdots\right), \ldots
\end{aligned} \tag{2.15}$$

Here S_1, S_2, ... correspond to the surface counter terms, which cancell the divergences in the bulk action and $M_4^\pm$ expresses the boundary at $t \to \pm\infty$.

Let us consider two copies of the de Sitter spaces $\mathrm{dS}_{(1)}$ and $\mathrm{dS}_{(2)}$. We also put one or two of the space-like branes, which can be regarded as boundaries connecting the two bulk de Sitter spaces, at finite t. Then if one takes the following action S instead of $S_{\mathrm{dS\,grav}}$,

$$S = S_{\mathrm{EH}} + S_{\mathrm{GH}} + 2S_1 = S_{\mathrm{dS\,grav}} + S_1 - S_2 - \cdots, \tag{2.16}$$

we obtain the following boundary theory in terms of the partition function:

$$\begin{aligned}\int_{\mathrm{dS}_5^{(1)}+\mathrm{dS}_5^{(1)}+M_4^++M_4^-}[dg][d\varphi]\mathrm{e}^{-S} &= \left(\int_{\mathrm{dS}_5}[dg][d\varphi]\mathrm{e}^{-S_{\mathrm{EH}}-S_{\mathrm{GH}}-S_1}\right)^2 \\ &= \mathrm{e}^{2S_2+\cdots}\left(\int_{\mathrm{dS}_5}[dg][d\varphi]\mathrm{e}^{-S_{\mathrm{grav}}}\right)^2 = \mathrm{e}^{-2W_{\mathrm{CFT}}+2S_2+\cdots}\,.\end{aligned} \tag{2.17}$$

Since S_2 can be regarded as the Einstein-Hilbert action on the boundary, the gravity on the boundary becomes dynamical. In other words, there is strong indication that our brane-world model at some conditions may be effectively described by 4d gravity interacting with some gauge theory.

Now we consider the quantum effects ($b' \neq 0$ case) on the brane in (2.13). Let us define a function $F(R^2)$ as follows:

$$F(R^2) = \frac{1}{\pi G}\left(\frac{1}{R}\sqrt{1-\frac{R^2}{l^2}}-\frac{1}{l}\right)R^4\,. \tag{2.18}$$

Then one can easily find

$$\begin{aligned}&F(0) = F\left(\frac{l^2}{2}\right) = 0\,, \qquad F(l^2) = -\frac{l^3}{\pi G}\,, \\ &F(R^2)\begin{cases} > 0 & \text{when } 0 < R^2 < \frac{l^2}{2} \\ < 0 & \text{when } \frac{l^2}{2} < R^2 \leq l^2 \end{cases}\,.\end{aligned} \tag{2.19}$$

The function $F(R^2)$ has a maximum

$$F = F_m \equiv \frac{l^3}{16\pi G}\left(-26+35\sqrt{1-\frac{9}{50}}\right) \tag{2.20}$$

when

$$R^2 = R_m^2 \equiv \frac{5l^2}{4}\left(1-\sqrt{1-\frac{9}{50}}\right) < \frac{l^2}{2}\,. \tag{2.21}$$

The above results tell

1. When $-8b' > F_m$ or $-8b' < -l^3/\pi G$, there is no solution in eq. (2.13). That is, the quantum effect exhibits the creation of the inflationary brane world.

2. When $0 < -8b' < F_m$, there appear two solutions in (2.13). The solution with larger radius R corresponds to the classical one in (2.14) but the radius R in the solution is smaller then that in the classical one. In other words, quantum effects try to prevent inflation. The solution with smaller radius can be regarded as the solution generated by only quantum effects on the brane. Anyway the radii R in both of the solutions are smaller than that in the classical one (2.14). Since $1/R$ corresponds to the rate of the expansion of the universe when S_4 is Wick-rotated into 4d de Sitter space, the quantum effect makes the rate larger.

3. When $0 > -8b' > -l^3/\pi G$, which is rather exotic case since usualy b' is negative as in (2.5), eq. (2.13) has unique solution corresponding to the solution in the classical case (2.14) and the quantum effect on the brane makes the radius R larger.

The de Sitter brane may be thought as inflationary brane. The natural question then appears how such inflation may become instable? The answer goes in the same way as in anomaly-driven inflation [17]. Despite the fact that the term in the effective action related with coefficient b'' does not give contribution to the equations of the motion, it is important in the study of perturbations. It may be shown, by analogy with [17, 18] that there is some bound for this coefficient which makes the inflation to be instable. Indeed, we got the alternative description of the brane-world as some 4d gravity with matter. Then, the analysis of instability of inflation may be repeated in all details and values of parameter b'' which ensure the instability may be found. We will not go to the details of such analysis as it repeats very much the same done recently in [17]. So principal possibility of the end of brane inflation exists.

If we Wick-rotate S_4 into the lorentzian signature, we can obtain 4d de Sitter space. In some choice of the time coordinate, the de Sitter space can be regarded as an inflationary universe. The rate of the inflation corresponds to the inverse of the radius of S_4. Hence, we estimated the role of quantum effects to creation of de Sitter brane-world. As one can see the brane inflation occurs on classical as well as on quantum levels in 5d de Sitter bulk space. Quantum effects not only decrease the radius of classically created de Sitter brane but also can produce another (purely quantum) de Sitter brane.

Let us give some remarks about the Wick-rotation of the above obtained brane solution. There are several ways for the Wick-rotation of the sphere into de Sitter space. The metric of S_5 can be expressed as

$$ds_{S_5}^2 = l^2\left(d\chi^2 + \sin^2\chi\left(d\eta^2 + \sin^2 d\Omega_2^2\right)\right) . \tag{2.22}$$

Here $d\Omega_2^2$ is the metric of 2 dimensional sphere. The brane S_4 can be embedded into S_5 by putting the coordinate χ to be a constant: $\chi = \chi_0$. Then the metric of S_4 has the folowing form

$$ds_{S_4}^2 = l^2\sin^2\chi_0\left(d\eta^2 + \sin^2 d\Omega_2^2\right) . \tag{2.23}$$

If we further write $d\Omega_2^2$ as

$$d\Omega_2^2 = d\theta^2 + \sin^2\theta d\phi^2 \tag{2.24}$$

and Wick-rotate ϕ by

$$\phi \to it_1 , \tag{2.25}$$

we obtain the static 4d de Sitter brane in static 5d de Sitter bulk space:

$$\begin{aligned} ds_{S_5}^2 &\to l^2\left(d\chi^2 + \sin^2\chi\left(d\eta^2 + \sin^2\eta\left(d\theta^2 - \sin^2\theta dt_1^2\right)\right)\right) , \\ ds_{S_4}^2 &\to l^2\sin^2\chi_0\left(d\eta^2 + \sin^2\eta\left(d\theta^2 - \sin^2\theta dt_1^2\right)\right) . \end{aligned} \tag{2.26}$$

On the other hand, if we Wick-rotate the coordinate χ by

$$\chi \to \frac{\pi}{2} + it_2 , \qquad \chi_0 \to \frac{\pi}{2} + it_0 , \tag{2.27}$$

the brane becomes the space-like surface of S_4 in 5d de Sitter space, which can be regarded as the inflationary universe

$$\begin{aligned} ds^2_{\mathrm{S}_5} &\to l^2\left(-dt_2^2+\cosh^2 t_2\left(d\eta^2+\sin^2\eta\Omega_2^2\right)\right)\,, \\ ds^2_{\mathrm{S}_4} &\to l^2\cosh^2 t_0\left(d\eta^2+\sin^2\eta\Omega_2^2\right)\,. \end{aligned} \tag{2.28}$$

When we Wick-rotate the coordinate η by

$$\eta\to\frac{\pi}{2}+it_3\,, \tag{2.29}$$

we obtain 4d de Sitter brane, which can be regarded as the inflationary universe

$$\begin{aligned} ds^2_{\mathrm{S}_5} &\to l^2\left(-\sin^2\chi dt_3^2+d\chi^2+\sin^2\chi\cosh^2 t_3 d\Omega_3^2\right)\,, \\ ds^2_{\mathrm{S}_4} &\to l^2\sin^2\chi_0\left(-dt_3^2+\cosh^2 t_3 d\Omega_3^2\right)\,. \end{aligned} \tag{2.30}$$

Here $d\Omega_3^2$ is the metric of the 3d unit sphere. The expression for 5d de Sitter bulk space is not so conventional.

In [17], trace anomaly driven inflationary model [15] has been studied using AdS/CFT correspondence. The brane, which is euclidean 4 sphere, can be nucleated due to quantum effects in AdS spacetime and it can be regarded as an instanton [3, 4]. The brane can be analytically continued into the lorentzian signature and the de Sitter space is nucleated. The 4d de Sitter space can be identified with the inflationary universe [15]. In [17], as in the original model in [15], it was shown that the de Sitter space is instable and decays into the matter dominant FRW universe. For such a decay, the term with coefficient b'' of $\mathcal{W}$ in (2.2) is important and affect the perturbation of the tensor part in the metric. Such term also appears as α' corrections in the string theory. Therefore this term maybe induced by the square of the scalar curvature even if on quantum level we took $b''=0$. In [17], following the arguments from [18], the inflation occurs until

$$T=t_*\sim -2\alpha(\gamma-1)R\,. \tag{2.31}$$

Here we can shoose $\alpha=\frac{16\pi^2}{N^2}b''$ when we consider $\mathcal{N}=4$ SU(N) Yang-Mills theory on the brane. The parameter γ is related with the initial perturbation from the de Sitter solution and can be expressed as the perturbation of the Hubble parameter H when the de Sitter universe is nucleated:

$$\gamma=\frac{1}{2}\ln\left(\frac{2H_0}{H_0-H}\right)\,. \tag{2.32}$$

Here H_0 is the inverse of R in (2.13), which is the radius of the 4d sphere. Thus, the arguments are presented which show the possibility to end the inflation.

3. FRW brane in 5d SdS black hole

We now consider the situation that radius depends on the "time" coordinate. Taking 5d Schwarzschild- de Sitter (SdS) black hole background, the obtained brane equation, which describes the dynamics of the brane, can be regarded as the induced Friedmann-Robertson-Walker (FRW) equation.

Starting with the Minkowski signature action one gets the following equation which generalizes the classical brane equation (2.12):

$$\begin{aligned}0 = &\frac{48l^4}{16\pi G}\left(A_{,z} - \frac{1}{l}\right)\mathrm{e}^{4A} + b'\left(4\partial_\tau^4 A + 16\partial_\tau^2 A\right) - \\ &- 4(b+b')\left(\partial_\tau^4 A - 2\partial_\tau^2 A - 6(\partial_\tau A)^2\partial_\tau^2 A\right). \end{aligned} \tag{3.1}$$

In (3.1), one uses the form of the metric as

$$ds^2 = dz^2 + \mathrm{e}^{2A(z,\tau)}\tilde{g}_{\mu\nu}dx^\mu dx^\nu\,, \qquad \tilde{g}_{\mu\nu}dx^\mu dx^\nu \equiv l^2\left(-d\tau^2 + d\Omega_3^2\right). \tag{3.2}$$

Here $d\Omega_3^2$ corresponds to the metric of 3 dimensional unit sphere. As a bulk space, one takes 5d Schwarzschild-de Sitter spacetime, whose metric is given by

$$ds^2_{\mathrm{dS-S}} = \frac{1}{h(a)}da^2 - h(a)dt^2 + a^2 d\Omega_3^2\,, \qquad h(a) = -\frac{a^2}{l^2} + 1 - \frac{16\pi GM}{3V_3 a^2}\,. \tag{3.3}$$

Here V_3 is the volume of the unit 3 sphere. If one chooses new coordinates (z,τ) by

$$\frac{\mathrm{e}^{2A}}{h(a)}A_{,z}^2 - h(a)t_{,z}^2 = 1\,, \qquad \frac{\mathrm{e}^{2A}}{h(a)}A_{,z}A_{,\tau} - h(a)t_{,z}t_{,\tau} = 0\,, \qquad \frac{\mathrm{e}^{2A}}{h(a)}A_{,\tau}^2 - h(a)t_{,\tau}^2 = -\mathrm{e}^{2A}\,. \tag{3.4}$$

the metric takes the form (3.2). Here $a = l\mathrm{e}^A$. Furthermore choosing a coordinate $\tilde{t}$ by $d\tilde{t} = l\mathrm{e}^A d\tau$, the metric on the brane takes FRW form:

$$\mathrm{e}^{2A}\tilde{g}_{\mu\nu}dx^\mu dx^\nu = -d\tilde{t}^2 + l^2\mathrm{e}^{2A}d\Omega_3^2\,. \tag{3.5}$$

Solving eqs. (3.4), one gets

$$H^2 = A_{,z}^2 - h\mathrm{e}^{-2A} = A_{,z}^2 + \frac{1}{l^2} - \frac{1}{a^2} + \frac{16\pi GM}{3V_3 a^4}\,. \tag{3.6}$$

Here the Hubble constant H is defined by $H = dA/d\tilde{t}$. Then using the brane eq. (3.1), we obtain

$$H^2 = -\frac{1}{a^2} + \frac{8\pi G_4 \rho}{3} \tag{3.7}$$

$$\begin{aligned}\rho = \frac{l}{a}\Bigg[\frac{M}{V_3 a^3} + \frac{3a}{16\pi G}\Bigg[\Bigg[\frac{1}{l} + \frac{\pi G}{3}\Bigg\{&-4b'\Big((H_{,\tilde{t}\tilde{t}\tilde{t}} + 4H_{,\tilde{t}}^2 + 7HH_{,\tilde{t}\tilde{t}} + 18H^2H_{,\tilde{t}} + 6H^4) + \\ &+ \frac{4}{a^2}(H_{,\tilde{t}} + H^2)\Big) + 4(b+b')\Big((H_{,\tilde{t}\tilde{t}\tilde{t}} + 4H_{,\tilde{t}}^2 + \\ &+ 7HH_{,\tilde{t}\tilde{t}} + 12H^2H_{,\tilde{t}}) - \frac{2}{a^2}(H_{,\tilde{t}} + H^2)\Big)\Bigg\}\Bigg]^2 + \frac{1}{l^2}\Bigg]\Bigg].\end{aligned} \tag{3.8}$$

This can be regarded as the quantum FRW equation of the brane universe. It again admits quantum-corrected dS brane solutions. Here 4d Newton constant G_4 is given by

$$G_4 = \frac{2G}{l}\,. \tag{3.9}$$

Note that forgetting about quantum corrections we have just standard FRW equation with some energy density ρ expressed in terms of 5d parameters:

$$\rho_c = \frac{Ml}{V_3 a^4} + \frac{3}{8\pi G l}\,, \tag{3.10}$$

where, from the point of view of 4d spacetime, the first term can be regarded as the contribution from the radiation and the second term as that from the cosmological constant. Since the energy density ρ in (3.7) contains the higher derivative terms, the quantum correction becomes important when the size of the universe changes rapidly, as in the early stage of the universe.

In a sense, brane FRW approach represents the attempt to describe the 4d cosmology in terms of the observer who knows about our 5d brane-world. As we already showed in the specific case of the previous section, these FRW equations may lead to reasonable early time cosmological behaviour.

Note that matter content may be chosen in such a way that brane CFT is dual to one of bulk SdS BH backgrounds in accord with dS/CFT correspondence. Further by differentiating eq. (3.7) with respect to $\tilde{t}$, one obtains the second FRW equation

$$H_{,\tilde{t}} = \frac{1}{a^2} - 4\pi G_4(\rho + p) \tag{3.11}$$

$$\begin{aligned}\rho + p = \frac{l}{a}\Bigg[\frac{4M}{3V_3 a^3} - \frac{1}{24 l^3 H}\Bigg[\frac{1}{l} + \frac{\pi G}{3}\Bigg\{&-4b'\Big((H_{,\tilde{t}\tilde{t}\tilde{t}} + 4H_{,\tilde{t}}^2 + 7HH_{,\tilde{t}\tilde{t}} + 18H^2 H_{,\tilde{t}} + 6H^4) + \\ &+ \frac{4}{a^2}(H_{,\tilde{t}} + H^2)\Big) + 4(b+b')\Big((H_{,\tilde{t}\tilde{t}\tilde{t}} + 4H_{,\tilde{t}}^2 + \\ &+ 7HH_{,\tilde{t}\tilde{t}} + 12H^2 H_{,\tilde{t}}) - \frac{2}{a^2}(H_{,\tilde{t}} + H^2)\Big)\Bigg\}\Bigg] \times \\ \times \Bigg\{-4b'\Big((H_{,\tilde{t}\tilde{t}\tilde{t}\tilde{t}} + 15H_{,\tilde{t}}H_{\tilde{t}\tilde{t}} + 7HH_{,\tilde{t}\tilde{t}\tilde{t}} + 18H^2 H_{,\tilde{t}\tilde{t}} + 36HH_{,\tilde{t}}^2 + 24H^3 H_{,\tilde{t}}) + \\ + \frac{4}{a^2}(H_{,\tilde{t}\tilde{t}} - 2H^3)\Big) + 4(b+b')\Big((H_{,\tilde{t}\tilde{t}\tilde{t}\tilde{t}} + 15H_{,\tilde{t}}H_{,\tilde{t}\tilde{t}} + \\ + 7HH_{,\tilde{t}\tilde{t}\tilde{t}} + 12H^2 H_{,\tilde{t}\tilde{t}} + 24HH_{,\tilde{t}}^2) - \frac{2}{a^2}(H_{,\tilde{t}\tilde{t}} - 2H^2)\Big)\Bigg\}\Bigg]\,.\end{aligned} \tag{3.12}$$

The quantum corrections from CFT are included into the definition of energy (pressure). These quantum corrected FRW equations are written from quantum-induced brane-world perspective.

The Schwarzschild-de Sitter black hole solution in (3.3) has a horizon at $a = a_H$, where $h(a)$ vanisif the higher derivative of the Hubble constant H is large, the quantum correction becomes essential.

The Schwarzschild-de Sitter black hole solution in (3.3) has a horizon at $a = a_H$, where $h(a)$ vanishes:

$$h(a_H) = -\frac{a_H^2}{l^2} + 1 - \frac{16\pi G M}{3V_3 a_H^2} = 0\,. \tag{3.13}$$

The solutions of eq. (3.13) are given by

$$a_{H\pm}^2 = \frac{l^2 \pm \sqrt{l^4 - 4\mu l^2}}{2}, \qquad \mu \equiv \frac{16\pi GM}{3V_3} \tag{3.14}$$

if

$$4\mu \leq l^2 . \tag{3.15}$$

The cosmological horizon lies at $a = a_{H+}$ and the black hole one at $a = a_{H-}$. When

$$4\mu = l^2 , \tag{3.16}$$

we have the extremal solution or Nariai space, where the horizons coincide with each other.

We now consider the case that the brane is static. Then since $H = 0$, the FRW equation (3.7) has the following form:

$$0 = \frac{2}{l^2} - \frac{1}{a^2} + \frac{\mu}{a^4} , \tag{3.17}$$

which has solutions

$$a_{0\pm} = \frac{l^2 \pm \sqrt{l^4 - 8\mu l^2}}{4} \tag{3.18}$$

if

$$8\mu \leq l^2 . \tag{3.19}$$

Since

$$a_{H-}^2 < a_{0\pm}^2 < a_{H+}^2 , \tag{3.20}$$

the brane can exist between the black hole horizon and the cosmological one. In (3.18), a_{0+} corresponds to the solution in (2.14) since the solution does not vanish even if $\mu = 0$. This tells the solution corresponding to a_{0+} is stable. Then the solution corresponding to a_{0-} should be unstable.

The brane FRW like equations in (3.7), (3.11) are rather different from the corresponding equations obtained in this frame in AdS/CFT correspondence [11] (see also [12]). In situation without quantum corrections on the brane one gets

$$H^2 = -\frac{1}{a^2} + \frac{8\pi G_4 \rho}{3} , \qquad \rho = \frac{l}{a}\left[\frac{M}{V_3 a^3} + \frac{3a}{8\pi G l^2}\right] , \tag{3.21}$$

$$H_{,\tilde{t}} = \frac{1}{a^2} - 4\pi G_4 (\rho + p) , \qquad \rho + p = \frac{l}{a} \cdot \frac{4M}{3V_3 a^3} . \tag{3.22}$$

In [11], the second term in ρ did not appear. Furthermore, we have

$$\rho + 3p = -\frac{3}{2\pi G l} \neq 0 , \tag{3.23}$$

which tells that the matter on the brane would not be conformal. Then the relation of cosmological entropy with Cardy formula [14] is not very clear. The difference λ of ρ from the AdS/CFT case is a constant

$$\lambda = \rho - \frac{l}{a} \cdot \frac{M}{V_3 a^3} = \frac{3}{8\pi G l} . \tag{3.24}$$

which indicates that the effective cosmological constant on the brane does not vanish. It would be really interesting to investigate this question in order to understand if the possibility to obtain the cosmological entropy bounds (so-called Cardy-Verlinde formula [13]) exists in the present (de Sitter brane-world) context.

4. Discussion

In summary, we discussed two 5d de Sitter bulk spaces (in different coordinate systems) connected by 4d de Sitter boundary playing the role of inflationary Universe. It is demonstrated that even in situation when brane tension is fixed by holographic RG the possibility of creation of such brane-world is not zero, the radius of de Sitter brane may be defined (unlike to the case of two 5d AdS bulks [3, 4]). Taking into account quantum brane fields via corresponding trace anomaly-induced effective action we proved the possibility of quantum creation of de Sitter brane-world. The role of quantum effects is to decrease classical de Sitter brane radius, as well as to induce purely quantum de Sitter brane with even smaller radius. It is important to note that brane CFT may be chosen to be dual to one of de Sitter bulk spaces which may be relevant for relation of brane-world approach with dS/CFT correspondence. This dS/CFT correspondence plays the important role in the demonstration that 4d gravity is localized and hence 4d inflationary brane may be described as some variant of anomaly-driven inflation (with the possibility to end the inflation). Finally, we considered moving (time-dependent) de Sitter brane in 5d SdS BH when quantum brane fields again contribute to effective action. The quantum creation of 4d de Sitter Universe is again possible. Using special system of coordinates where brane equations look like FRW equations the comparison of such (quantum corrected) FRW-like equations in SdS BH bulk with the ones in SAdS BH is done.

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